

Back-reaction instabilities of relativistic cosmic rays

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ABSTRACT

We explore in the multi-fluid approach streaming instabilities of the electron-ion plasma with relativistic and ultra-relativistic cosmic rays in the background magnetic field. Cosmic rays can be both electrons and protons. The drift speed of cosmic rays is directed along the magnetic field. In equilibrium, the return current of the background plasma is taken into account. One-dimensional perturbations parallel to the magnetic field are considered. The dispersion relations are derived for transverse and longitudinal perturbations. It is shown that the back-reaction of magnetized cosmic rays generates a new instability with the growth rate much larger than that for the Bell instability, while for unmagnetized cosmic rays, the growth rate is analogous to the Bell's. Some difference between two models of the return current in equilibrium is demonstrated. For longitudinal perturbations, an instability is found in the case of ultra-relativistic cosmic rays. The results obtained can be applied for investigations of astrophysical objects such as supernova remnant shocks, galaxy clusters, intracluster medium, and so on.

Key words: instabilities - magnetic fields - plasmas - waves - cosmic rays - galaxies:clusters:general

1. INTRODUCTION

It is known for a long time that a return current arises in a plasma penetrated by an external beam current, which is nearly equal to the imposed beam current (Roberts & Bennett 1968). A theory of this phenomena for the laboratory plasma has been developed in a number of early papers (Roberts & Bennett 1968; Cox & Bennett 1970; Hammer & Rostoker 1970; Lee & Sudan 1971; Berk & Pearlstein 1976). It was shown that the induced plasma current depends on the spatio-temporal shape of the imposed current and is transferred by plasma species. For external currents of the cylindrical shape, it has been found that the return current lies almost entirely within the beam channel (Cox & Bennett 1970; Hammer & Rostoker 1970; Lee & Sudan 1971). In the bounded magnetized plasma with the given nonstationary sheet current, the return current can change with time and be not equal to the external current (Berk & Pearlstein 1976). However, inclusion of the surface current in the perfectly conducting walls results in the full compensation of both currents.

The return currents in astrophysics are considered for media where cosmic rays are present. It is assumed that in equilibrium the total current of cosmic rays and plasma is equal to zero. The models are explored in which the equilibrium current is directed along (e.g. Achterberg 1983; Zweibel 2003; Bell 2004) and across (Riquelme & Spitkovsky 2010; Nekrasov & Shadmehri 2012) the background magnetic field. In the case of currents parallel to the magnetic field, one considers the three-component medium, the electrons, ions, and cosmic rays, where each species has the own drift velocity (Achterberg 1983), as well as a four-component one. In the last case, one assumes that the background plasma has no drift velocities while cosmic rays and an additional electron component (for the proton cosmic rays) drift together (Zweibel 2003; Bell 2004; Zweibel & Everett 2010).

The kinetic consideration of cosmic rays drifting along the magnetic field has been

provided by Achterberg (1983), Zweibel (2003), Bell (2004), and Reville et al. (2007) for perturbations also parallel to the magnetic field. The well-known non-resonant Bell instability (Bell 2004) has the large growth rate for perturbation wavelengths shorter than the mean Larmor radius of cosmic ray protons defined by their longitudinal momentum. In this case, the contribution of cosmic rays to the kinetic dispersion relation is small (Zweibel 2003; Bell 2004). Thus, the back-reaction of cosmic rays is absent in unstable short-wavelength perturbations mentioned above. In the opposite case of long-wavelength perturbations, the perturbed currents of cosmic rays (protons) and additional electrons compensate each other, if only the electric drift of particles is taken into account (Zweibel 2003).

However, involving the polarizational current of cosmic rays in the multi-fluid approach is also important in the derivation of the dispersion relation for cosmic ray streaming instabilities. This effect was not considered e.g. by Zweibel (2003) and Bell (2004). As we show here, the back-reaction of magnetized cosmic rays gives rise to streaming instabilities which are different from that found by Bell (2004).

In the present paper, we investigate streaming instabilities of the electron-ion plasma with cosmic rays up to ultra-relativistic energies in the background magnetic field, using the multi-fluid approach. We assume that cosmic rays, which can be both protons and electrons, drift along the magnetic field. One-dimensional perturbations also parallel to the magnetic field are treated. In this case, transverse and longitudinal movements are split. For generality, we take into account the thermal energy exchange between background electrons and ions and electron thermal conductivity. We derive dispersion relations for the transverse and longitudinal perturbations. For the first case, the two models with three and four components described above are used and corresponding results are compared. Analogous consideration of these models for shocks has been provided by Amato & Blasi

(2009). New instabilities due to the back-reaction of cosmic rays are found.

The paper is organized as follows. Section 2 contains the fundamental equations for plasma, cosmic rays, and electromagnetic fields. Equilibrium state is discussed in Section 3. In Section 4, the transverse perturbations with magnetized and unmagnetized cosmic rays are explored. We investigate longitudinal perturbations in Section 5. In Section 6, we discuss results obtained in the preceding sections. Conclusive remarks are given Section 7.

2. BASIC EQUATION FOR PLASMA AND COSMIC RAYS

The fundamental equations for the plasma that we consider here are the following:

$$\frac{\partial \mathbf{v}_j}{\partial t} + \mathbf{v}_j \cdot \nabla \mathbf{v}_j = -\frac{\nabla p_j}{m_j n_j} + \frac{q_j}{m_j} \mathbf{E} + \frac{q_j}{m_j c} \mathbf{v}_j \times \mathbf{B}, \quad (1)$$

the equation of motion,

$$\frac{\partial n_j}{\partial t} + \nabla \cdot n_j \mathbf{v}_j = 0, \quad (2)$$

the continuity equation,

$$\frac{\partial T_i}{\partial t} + \mathbf{v}_i \cdot \nabla T_i + (\gamma - 1) T_i \nabla \cdot \mathbf{v}_i = \nu_{ie}^\varepsilon(n_e, T_e) (T_e - T_i) \quad (3)$$

and

$$\frac{\partial T_e}{\partial t} + \mathbf{v}_e \cdot \nabla T_e + (\gamma - 1) T_e \nabla \cdot \mathbf{v}_e = -(\gamma - 1) \frac{1}{n_e} \nabla \cdot \mathbf{q}_e - \nu_{ei}^\varepsilon(n_i, T_e) (T_e - T_i) \quad (4)$$

are the temperature equations for ions and electrons. In Equations (1) and (2), the subscript $j = i, e$ denotes the ions and electrons, respectively. Notations in Equations (1)-(4) are the following: q_j and m_j are the charge and mass of species $j = i, e$, \mathbf{v}_j is the hydrodynamic velocity, n_j is the number density, $p_j = n_j T_j$ is the thermal pressure, T_j is the temperature, $\nu_{ie}^\varepsilon(n_e, T_e)$ ($\nu_{ei}^\varepsilon(n_i, T_e)$) is the frequency of the thermal energy exchange between ions (electrons) and electrons (ions) being $\nu_{ie}^\varepsilon(n_e, T_e) = 2\nu_{ie}$, where ν_{ie} is the collision frequency

of ions with electrons (Braginskii 1965), $n_i \nu_{ie}^\varepsilon(n_e, T_e) = n_e \nu_{ei}^\varepsilon(n_i, T_e)$, γ is the ratio of the specific heats, \mathbf{E} and \mathbf{B} are the electric and magnetic fields, and c is the speed of light in a vacuum. We do not take into account collisions between the ions and electrons in the momentum equation. However, the thermal exchange should be included because it must be compared with the dynamical time. The value \mathbf{q}_e in Equation (4) is the electron heat flux (Braginskii 1965). In a weakly collisional plasma which is here considered, the electron Larmor radius is much smaller than the electron collisional mean free path. In this case, the electron thermal flux is mainly directed along the magnetic field,

$$\mathbf{q}_e = -\chi_e \mathbf{b} (\mathbf{b} \cdot \nabla) T_e, \quad (5)$$

where χ_e is the electron thermal conductivity coefficient and $\mathbf{b} = \mathbf{B}/B$ is the unit vector along the magnetic field. We assume that the thermal flux in equilibrium is absent.

Equations for relativistic cosmic rays we take in the form (e.g. Lontano, Bulanov & Koga 2002)

$$\frac{\partial (R_{cr} \mathbf{p}_{cr})}{\partial t} + \mathbf{v}_{cr} \cdot \nabla (R_{cr} \mathbf{p}_{cr}) = -\frac{\nabla p_{cr}}{n_{cr}} + q_{cr} \left(\mathbf{E} + \frac{1}{c} \mathbf{v}_{cr} \times \mathbf{B} \right), \quad (6)$$

$$\left(\frac{\partial}{\partial t} + \mathbf{v}_{cr} \cdot \nabla \right) \left(\frac{p_{cr} \Gamma_{cr}^{\Gamma_{cr}}}{n_{cr}^{\Gamma_{cr}}} \right) = 0, \quad (7)$$

where

$$R_{cr} = 1 + \frac{\Gamma_{cr}}{\Gamma_{cr} - 1} \frac{T_{cr}}{m_{cr} c^2}. \quad (8)$$

In these equations, $\mathbf{p}_{cr} = \gamma_{cr} m_{cr} \mathbf{v}_{cr}$ is the momentum of a cosmic ray particle having the rest mass m_{cr} and velocity \mathbf{v}_{cr} , q_{cr} is the charge, $p_{cr} = \gamma_{cr}^{-1} n_{cr} T_{cr}$ is the kinetic pressure, n_{cr} is the number density in the laboratory frame, Γ_{cr} is the adiabatic index, $\gamma_{cr} = (1 - \mathbf{v}_{cr}^2/c^2)^{-1/2}$ is the relativistic factor. The continuity equation is the same as Equation (2) for $j = cr$. Equation (8) can be used for both cold non-relativistic, $T_{cr} \ll m_{cr} c^2$, and hot relativistic, $T_{cr} \gg m_{cr} c^2$, cosmic rays. In the first (second) case, we have $\Gamma_{cr} = 5/3$ ($4/3$) (Lontano,

Bulanov & Koga 2002). The general form of the value R_{cr} applying at any relations between T_{cr} and $m_{cr}c^2$, can be found e.g. in Toepfer (1971) and Dzhavakhishvili & Tsintsadze (1973).

Equations (1)-(4), (6), and (7) are solved together with Maxwell's equations

$$\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} \quad (9)$$

and

$$\nabla \times \mathbf{B} = \frac{4\pi}{c} \mathbf{j} + \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t}, \quad (10)$$

where $\mathbf{j} = \mathbf{j}_{pl} + \mathbf{j}_{cr} = \sum_j q_j n_j \mathbf{v}_j + \mathbf{j}_{cr}$.

3. EQUILIBRIUM STATE

We will consider a uniform plasma embedded in the uniform magnetic field \mathbf{B}_0 (subscript 0 here and below denotes background parameters) directed along the z -axis. We assume that in equilibrium the plasma is penetrated by a uniform beam of cosmic rays having the uniform streaming velocity v_{cr0} along the z -axis. The reverse plasma current along this axis compensating the current of cosmic rays is provided by the streaming velocities of electrons, v_{e0} , and ions, v_{i0} . The quasi-neutrality is satisfied due to cosmic ray charge neutralizaion from the background environment (Alfvén 1939). Thus, we have two equations in equilibrium

$$q_e n_{e0} v_{e0} + q_i n_{i0} v_{i0} + q_{cr} n_{cr0} v_{cr0} = 0 \quad (11)$$

and

$$q_e n_{e0} + q_i n_{i0} + q_{cr} n_{cr0} = 0. \quad (12)$$

Such a three-component model corresponds to the one considered by Achterberg (1983). In papers by Zweibel (2003) and Bell (2004), the four-component model has been

explored in which the additional electrons (in the case of the proton cosmic rays) drift with the cosmic ray drift velocity. We show below that there is some difference between these two models.

4. TRANSVERSE PERTURBATIONS

We will treat one-dimensional perturbations along the background magnetic field. From Equations (9) and (10), it is followed that in this case the transverse and longitudinal perturbations are split. The transverse wave equations have the form

$$\begin{aligned} c^2 \left(\frac{\partial}{\partial t} \right)^{-2} \frac{\partial^2 E_{1x}}{\partial z^2} &= 4\pi \left(\frac{\partial}{\partial t} \right)^{-1} j_{1x} + E_{1x}, \\ c^2 \left(\frac{\partial}{\partial t} \right)^{-2} \frac{\partial^2 E_{1y}}{\partial z^2} &= 4\pi \left(\frac{\partial}{\partial t} \right)^{-1} j_{1y} + E_{1y}. \end{aligned} \quad (13)$$

The perturbed currents $j_{1x,y}$ are given by Equations (A16), (A17), (B10), and (B11). Substituting them into Equation (13), we obtain

$$\left[c^2 \frac{\partial^2}{\partial z^2} \left(\frac{\partial}{\partial t} \right)^{-2} - \varepsilon_{xx} - 1 \right]^2 E_{1x,y} + \varepsilon_{xy}^2 E_{1x,y} = 0, \quad (14)$$

where

$$\varepsilon_{xx} = \varepsilon_{plx} + \varepsilon_{crx}, \quad (15)$$

$$\varepsilon_{xy} = \varepsilon_{plx} + \varepsilon_{crx}.$$

For perturbations of the form $\exp(ik_z - i\omega t)$, we find from Equation (14) the dispersion relation

$$\frac{k_z^2 c^2}{\omega^2} - \varepsilon_{xx} - 1 = \pm i \varepsilon_{xy}. \quad (16)$$

4.1. Magnetized Species

We first consider Equation (16) in the case in which all species are magnetized, i.e.

$$\begin{aligned}\omega_{cj}^2 &\gg D_{tj}^2, \\ \omega_{ccr}^2 &\gg D_{cr}^2.\end{aligned}\tag{17}$$

Using Equation (17), we calculate the values $\varepsilon_{plx,y}$ and $\varepsilon_{crx,y}$ given by Equations (A17) and (B11), respectively, and substitute them into Equation (15). Then from Equation (16), we derive the following dispersion relation:

$$k_z^2 c^2 + \sum_j \frac{\omega_{pj}^2 D_{tj}^2}{\omega_{cj}^2} + \gamma_{cr0} R_{cr0} \frac{\omega_{pcr}^2 D_{tcr}^2}{\omega_{ccr}^2} = \mp i \left(\sum_j \frac{\omega_{pj}^2 D_{tj}}{\omega_{cj}} + \frac{\omega_{pcr}^2 D_{tcr}}{\omega_{ccr}} \right),\tag{18}$$

where $D_{tj,cr} = -i\omega + ik_z v_{j,cr0}$. In Equation (18), we have neglected the contribution of the displacement current and small terms proportional to $D_{tj,cr}^3/\omega_{cj,cr}^3$. According to Equations (11) and (12), the right-hand side of Equation (18) is equal to zero. Thus, we obtain

$$\alpha_e (\omega - k_z v_{e0})^2 + \alpha_i (\omega - k_z v_{i0})^2 + \alpha_{cr} (\omega - k_z v_{cr0})^2 = k_z^2 c^2,\tag{19}$$

where $\alpha_j = \omega_{pj}^2/\omega_{cj}^2$ and $\alpha_{cr} = \gamma_{cr0} R_{cr0} \omega_{pcr}^2/\omega_{ccr}^2$. Solution of Equation (19) is given by

$$\omega = \frac{A_2}{A_1} k_z \pm \frac{1}{A_1} k_z (A_2^2 - A_1 A_3)^{1/2},\tag{20}$$

where

$$A_1 = \alpha_e + \alpha_i + \alpha_{cr},\tag{21}$$

$$A_2 = \alpha_e v_{e0} + \alpha_i v_{i0} + \alpha_{cr} v_{cr0},$$

$$A_3 = \alpha_e v_{e0}^2 + \alpha_i v_{i0}^2 + \alpha_{cr} v_{cr0}^2 - c^2.$$

Using Equation (21), we find the expression $A_2^2 - A_1 A_3$

$$A_2^2 - A_1 A_3 = A_1 c^2 - \alpha_e \alpha_i (v_{e0} - v_{i0})^2 - \alpha_e \alpha_{cr} (v_{cr0} - v_{e0})^2 - \alpha_i \alpha_{cr} (v_{cr0} - v_{i0})^2.\tag{22}$$

Equation (20) describes the streaming instability if $(A_2^2 - A_1 A_3) < 0$.

The number density of cosmic rays is considerably smaller than the density of the background plasma. Therefore, we can conclude from Equation (11) that $v_{e,i0} \ll v_{cr0}$. In this case, Equation (22) can be written in the form

$$A_2^2 - A_1 A_3 \simeq (\alpha_i + \alpha_{cr}) c^2 - \alpha_i \alpha_{cr} v_{cr0}^2. \quad (23)$$

The growth rate of instability $\delta = \text{Im} \omega$ found from Equation (20) in the case $\alpha_i \alpha_{cr} v_{cr0}^2 \gg (\alpha_i + \alpha_{cr}) c^2$ is equal to

$$\delta = \frac{(\alpha_i \alpha_{cr})^{1/2}}{\alpha_i + \alpha_{cr}} k_z v_{cr0}. \quad (24)$$

This new instability arises due to the cosmic ray back-reaction, i.e. due to the same dynamics of cosmic rays as that of the plasma connected with the polarizational drift of the species.

For the four-component model consisting of the background ions and electrons without drift velocities, proton cosmic rays, and additional electrons with the cosmic ray number density and drift velocity (Zweibel 2003), Equation (18) has the solution

$$\omega = \frac{\alpha_{cr}}{\alpha_i + \alpha_{cr}} k_z v_{cr0} \pm \frac{k_z}{\alpha_i + \alpha_{cr}} \left[-\alpha_i \alpha_{cr} v_{cr0}^2 + (\alpha_i + \alpha_{cr}) c^2 \right]^{1/2},$$

which is analogous to solution given by Equation (20) (see Equations (21) and (23)) except for the phase velocity.

4.2. Unmagnetized Cosmic Rays

In this section, we assume that cosmic rays are unmagnetized

$$D_{cr}^2 \gg \omega_{ccr}^2. \quad (25)$$

This condition can be satisfied for the relativistic cosmic rays for which $\gamma_{cr0}R_{cr0} \gg 1$. Then, we obtain

$$\begin{aligned}\varepsilon_{crxx} &= -\frac{\omega_{pcr}^2}{\gamma_{cr0}R_{cr0}\omega^2}, \\ \varepsilon_{crxy} &= -\frac{\omega_{pcr}^2\omega_{ccr}}{\gamma_{cr0}R_{cr0}D_{cr}\omega^2}.\end{aligned}\tag{26}$$

The plasma ions and electrons stay magnetized. Substituting Equation (26) and $\varepsilon_{plx,y}$ into Equation(16), we will have

$$k_z^2 c^2 - \alpha_e (\omega - k_z v_{e0})^2 - \alpha_i (\omega - k_z v_{i0})^2 = \pm \beta_{cr} (\omega - k_z v_{cr0}),\tag{27}$$

where $\beta_{cr} = \omega_{pcr}^2/\omega_{ccr}$. When obtaining the right-hand side of this equation, we have used Equations (11) and (12). We note that Equation (27) does not contain the contribution of the cosmic ray perturbed dynamics which is small in comparison with the plasma current produced by the electric drift velocities of ions and electrons. Solution of Equation (27) is given by

$$\begin{aligned}\omega &= \frac{1}{\alpha_i} \left(\alpha_e k_z v_{e0} + \alpha_i k_z v_{i0} \mp \frac{\beta_{cr}}{2} \right) \\ &\pm \frac{1}{\alpha_i} \left[\pm \alpha_i \beta_{cr} k_z v_{cr0} - \alpha_e \alpha_i k_z^2 (v_{e0} - v_{i0})^2 + \frac{1}{4} \beta_{cr}^2 + \alpha_i k_z^2 c^2 \right]^{1/2}.\end{aligned}\tag{28}$$

From Equations (11) and (12), it is followed that $v_{e0} - v_{i0} \simeq (q_{cr}n_{cr0}/q_i n_{i0}) v_{cr0}$ ($q_i = -q_e$). An estimation of the ratio of the second term in the squared brackets in Equation (28) to the first one gives the value $(n_{cr0}/n_{i0}) (k_z v_{cr0}/\omega_{ce})$, which can be much less than unity.

Solution of the dispersion relation for the four-component medium considered above is the following:

$$\omega = \mp \frac{1}{2} \frac{\beta_{cr}}{\alpha_i} \pm \left(\pm \frac{\beta_{cr}}{\alpha_i} k_z v_{cr0} - \frac{\beta_{cr}}{\alpha_i |\omega_{ce}|} k_z^2 v_{cr0}^2 + \frac{1}{4} \frac{\beta_{cr}^2}{\alpha_i^2} + k_z^2 c_{Ai}^2 \right)^{1/2},\tag{29}$$

where $c_{Ai} = (B_0^2/4\pi n_{i0} m_i)^{1/2}$ is the ion Alfvén velocity and the sign $||$ denotes an absolute value. We see some differences between Equations (28) and (29).

Equation (29) applied to the proton cosmic rays coincides with Equation (8) in the paper by Zweibel & Everett (2010), if we neglect the term proportional to v_{cr0}^2 (assuming that $k_z v_{cr0} \ll |\omega_{ce}|$) and take the lower sign (see also Zweibel (2003) and Bell (2004)). This coincidence is due to the absence of the dynamical contribution of unmagnetized cosmic rays to the dispersion relation. However, the nature of this effect is different in both cases. In our one-dimensional case, the transverse perturbations of cosmic rays does not contain the thermal pressure. However from the kinetic treatment, it is followed that cosmic rays must be cold along the magnetic field to use the transverse magnetohydrodynamical equations. Cosmic rays are unmagnetized under condition given by Equation (25). At the same time, the kinetic consideration of cosmic rays also shows that cosmic rays are unmagnetized in perturbations with wavelengths much smaller than their Larmor radius defined by the thermal velocity along the magnetic field (Zweibel 2003; Bell 2004).

We note that if we set $v_{e,i0} = 0$ in Equation (28) (or on the left-hand side of Equation (27)), we return to Equation (29) without the term $\sim v_{cr0}^2$.

5. LONGITUDINAL PERTURBATIONS

5.1. Dispersion Relation

We now consider potential perturbations along the background magnetic field. The wave equation is the following (see Equation (10)):

$$4\pi \left(\frac{\partial}{\partial t} \right)^{-1} j_{1z} + E_{1z} = 0. \quad (30)$$

In Appendices A and B, there are obtained the plasma, j_{pl1z} , and cosmic ray, j_{cr1z} , perturbed currents (Equations (A18) and (B12), respectively). Substitution them into Equation (30)

and Fourier transformation lead to the dispersion relation

$$0 = \frac{D}{L} \left[\frac{\omega_{pe}^2}{D_{te}} \left(L_{1i} D_{te} - L_{2e} D_{ti} \frac{q_i m_e}{q_e m_i} \right) + \frac{\omega_{pi}^2}{D_{ti}} \left(L_{1e} D_{ti} - L_{2i} D_{te} \frac{q_e m_i}{q_i m_e} \right) \right] + \frac{\omega_{pcr}^2}{L_{cr}} + 1, \quad (31)$$

where $\partial/\partial t = -i\omega$ and $\partial/\partial z = ik_z$. This equation will be investigated in the limiting cases.

5.2. Cold Electrons and Ions

We first consider the cold plasma species for which

$$D_{tj}^2 \gg \frac{T_0}{m_j} k_z^2. \quad (32)$$

For cosmic rays, we here and below assume that the following condition is satisfied:

$$D_{tcr}^2 \gg \frac{\Gamma_{cr} T_{cr0}}{\gamma_{cr0}^4 E_{cr0} m_{cr}} k_z^2, \quad (33)$$

where

$$E_{cr0} = R_{cr0} - \frac{\Gamma_{cr} T_{cr0}}{m_{cr} c^2} \frac{v_{cr0}^2}{c^2}.$$

In this case, the first term on the right-hand side of Equation (B13) is dominant. We note that the temperature of cosmic rays can be relativistic, i.e. $T_{cr0}/m_{cr} c^2 \gg 1$. Then, using Equations (A6), (A8), (A10), (A12), and (B13) under conditions defined by Equations (32) and (33), we obtain Equation (31) in the form

$$\frac{\omega_{pe}^2}{(\omega - k_z v_{e0})^2} + \frac{\omega_{pi}^2}{(\omega - k_z v_{i0})^2} + \frac{\omega_{pcr}^2}{\gamma_{cr0}^3 E_{cr0} (\omega - k_z v_{cr0})^2} = 0, \quad (34)$$

where for simplicity we have neglected unity.

Let us consider the case in which the back-reaction of cosmic rays plays the role. Then, solution of Equation (34) will be the following:

$$\omega = k_z v_{cr0} \left(1 + i \gamma_{cr0}^{-3/2} E_{cr0}^{-1/2} \frac{\omega_{pcr}}{\omega_{pe}} \right). \quad (35)$$

If the cosmic ray back-reaction does not play the role, solution of Equation (34) will be given by

$$\omega = k_z v_{i0} + i \left(\frac{m_e}{m_i} \right)^{1/2} k_z |v_{i0} - v_{e0}|. \quad (36)$$

The ratio of the growth rate defined by Equation (36) to that of Equation (35) is equal to $\gamma_{cr0}^3 E_{cr0} (m_{cr}/m_i) (n_{cr0}/n_{i0})$. Thus, the back-reaction can result in considerably larger growth rate even at $\gamma_{cr0} \gg 1$ and $T_{cr} \gg m_{cr} c^2$.

5.3. Hot Electrons and Cold and Hot Ions

Consideration shows that in the cases $D_{te}^2 \ll (T_0/m_e) k_z^2, D_{ti}^2 \gg (T_0/m_i) k_z^2$ and $D_{ti}^2 \ll (T_0/m_i) k_z^2$ the frequency ω is of the order of $k_z v_{cr0}$ as that in Equation (35). Equation (33) results in condition $\gamma_{cr0} \gg 1$ when $v_{cr0} \simeq c$. Thus, the temperature of the background plasma should be relativistic. However, this contradicts the basic equations where plasma is non-relativistic. Therefore, conditions for hot plasma are invalid. Taking into account other terms in Equation (B13) does not give an instability.

6. DISCUSSION

We now discuss the growth rates and conditions of their derivation for transverse perturbations considered in Section 4. For magnetized cosmic rays obeying to Equation (17), the growth rate is given by Equation (24). Below, we assume that ions and cosmic rays are the protons. Let us consider first the case in which $\alpha_i \gg \alpha_{cr}$ or

$$1 \gg \gamma_{cr0} R_{cr0} \frac{n_{cr0}}{n_{i0}}. \quad (37)$$

Then, the condition of instability is the following:

$$\gamma_{cr0} R_{cr0} \frac{n_{cr0}}{n_{i0}} \gg \frac{c_{Ai}^2}{v_{cr0}^2} \quad (38)$$

(see Equation (23)). The growth rate is equal to

$$\delta = \left(\gamma_{cr0} R_{cr0} \frac{n_{cr0}}{n_{i0}} \right)^{1/2} k_z v_{cr0}. \quad (39)$$

This growth rate increases with the wave number k_z . Let us find its value for $k_z = k_{\text{Bell}}$, where

$$k_{\text{Bell}} = \frac{1}{2} \omega_{ci} \frac{n_{cr0}}{n_{i0}} \frac{v_{cr0}}{c_{Ai}^2}$$

is the wave number at which there is the fastest growing mode for the Bell instability (Bell 2004; Zweibel & Everett 2010). Then from Equation (39), we obtain

$$\frac{\delta(k_{\text{Bell}})}{\delta_{\text{Bell}}} = \left(\gamma_{cr0} R_{cr0} \frac{n_{cr0}}{n_{i0}} \right)^{1/2} \frac{v_{cr0}}{c_{Ai}}, \quad (40)$$

where

$$\delta_{\text{Bell}} = \frac{1}{2} \omega_{ci} \frac{n_{cr0}}{n_{i0}} \frac{v_{cr0}}{c_{Ai}}$$

is the maximal growth rate of the Bell instability (Bell 2004; Zweibel & Everett 2010).

According to Equation (38), it is followed from Equation (40) that $\delta(k_{\text{Bell}}) \gg \delta_{\text{Bell}}$. For the wave numbers such that

$$k_z \gg \frac{1}{2} \left(\frac{1}{\gamma_{cr0} R_{cr0}} \frac{n_{cr0}}{n_{i0}} \right)^{1/2} \frac{\omega_{ci}}{c_{Ai}},$$

the growth rate described by Equation (39) under conditions given by Equations (37) and (38) is much larger than the maximal growth rate of the Bell instability. From Equations (20) and (21), we see that $\text{Re } \omega \ll \delta$. The wave numbers are limited from above by condition $\omega_{ci}^2 / \gamma_{cr0}^2 R_{cr0}^2 v_{cr0}^2 \gg k_z^2$ (see Equation (17)). From here, we can obtain an estimation of the maximum of the growth rate given by Equation (39)

$$\delta_{\text{max}} \sim \omega_{ci} \left(\frac{1}{\gamma_{cr0} R_{cr0}} \frac{n_{cr0}}{n_{i0}} \right)^{1/2}.$$

The case $\alpha_{cr} \gg \alpha_i$ or

$$\gamma_{cr0} R_{cr0} \frac{n_{cr0}}{n_{i0}} \gg 1 \quad (41)$$

can be satisfied for ultra-relativistic cosmic rays for which $\gamma_{cr0} \gg 1$ and/or $R_{cr0} \gg 1$. In the last case, the temperature of cosmic rays is relativistic one, $T_{cr} \gg m_{cr} c^2$. The condition of instability has the form

$$v_{cr0}^2 \gg c_{Ai}^2. \quad (42)$$

The growth rate is given by

$$\delta = \left(\frac{1}{\gamma_{cr0} R_{cr0}} \frac{n_{i0}}{n_{cr0}} \right)^{1/2} k_z v_{cr0}. \quad (43)$$

A comparison of this growth rate with the Bell's one at $k_z = k_{\text{Bell}}$ gives

$$\frac{\delta(k_{\text{Bell}})}{\delta_{\text{Bell}}} = \left(\frac{1}{\gamma_{cr0} R_{cr0}} \frac{n_{i0}}{n_{cr0}} \right)^{1/2} \frac{v_{cr0}}{c_{Ai}}. \quad (44)$$

The first factor on the right-hand side of Equation (44) is less than unity and the second one is larger than unity (see Equations (41) and (42), respectively). Therefore, the relation between $\delta(k_{\text{Bell}})$ and δ_{Bell} depends on specific parameters. However, in the region

$$k_z \gg \frac{1}{2} (\gamma_{cr0} R_{cr0})^{1/2} \left(\frac{n_{cr0}}{n_{i0}} \right)^{3/2} \frac{\omega_{ci}}{c_{Ai}},$$

the growth rate given by Equation (43) is larger than the Bell growth rate. In the case under consideration, we have $\text{Re } \omega = k_z v_{cr0} \gg \delta$ (see Equations (20) and (21)). Thus, we find the upper limit for k_z^2

$$k_z^2 \ll \frac{\omega_{ci}^2}{v_{cr0}^2} \frac{1}{\gamma_{cr0} R_{cr0}} \frac{n_{cr0}}{n_{i0}}.$$

The maximal estimation of the growth rate given by Equation (43) is the following:

$$\delta_{\text{max}} \sim \frac{\omega_{ci}}{\gamma_{cr0} R_{cr0}}.$$

For unmagnetized cosmic rays satisfying Equation (25) and magnetized background electrons and ions, solution of the Equation (27) is given by Equation (28). In the case

$v_{cr0}^2/c_{Ai}^2 \gg 1$, the growth rate has the form δ_{Bell} (see above). The frequency ω is much smaller than $k_{\text{Bell}}v_{cr0}$. Thus, Equation (25) takes the form

$$\gamma_{cr0} R_{cr0} \frac{n_{cr0}}{n_{i0}} \frac{v_{cr0}^2}{c_{Ai}^2} \gg 1, \quad (45)$$

where we have inserted k_{Bell} . We note that under Equation (45) cosmic rays which are cold in the longitudinal direction do not contribute to the dispersion relation, i.e. the cosmic ray back-reaction is absent. We obtain for hot cosmic rays $p_{\parallel cr} \gg m_{cr}\omega_{ccr}/k_z$ an analogous result, where $p_{\parallel cr}$ is the average momentum along the magnetic field (Zweibel 2003; Bell 2004). Substitution to the last condition the value k_{Bell} gives

$$\frac{n_{cr0}}{n_{i0}} \frac{v_{cr0}}{c_{Ai}^2} \frac{p_{\parallel cr}}{m_{cr}} \gg 1,$$

where m_{cr} is the rest mass.

Let us discuss longitudinal perturbations. In the case of the cold background plasma expressed by Equation (32) and of Equation (33) for cosmic rays, solution of Equation (34) is given by Equation (35). Equation (32) can be written as $v_{cr0}^2 \gg T_0/m_e$. For cosmic rays, Equation (33) takes the form

$$\frac{n_{cr0}}{n_{e0}} \gg \frac{\Gamma_{cr} T_{cr0}}{\gamma_{cr0} m_e v_{cr0}^2}. \quad (46)$$

Equation (46) can be satisfied for ultra-relativistic cosmic rays with $v_{cr0} \simeq c$ and $\gamma_{cr0} \gg 1$. The growth rate δ is the following:

$$\delta = \gamma_{cr0}^{-3/2} E_{cr0}^{-1/2} \left(\frac{m_e}{m_{cr}} \frac{n_{cr0}}{n_{e0}} \right)^{1/2} k_z v_{cr0}.$$

This growth rate is much smaller than what is given, for example, by Equation (39).

The streaming instabilities driven by cosmic rays may play a significant role in such environments as the shocks caused by supernova remnants (Koyama et al. 1995; Allen et al. 1997; Tanimori et al. 1998; Vink & Laming 2003; Bell 2004), galaxy clusters (e.g. Brunetti et al. 2001; Pfrommer & Enßlin 2004), intracluster medium (e.g. Enßlin 2003; Guo & Oh

2008; Sharma et al. 2009; Sharma, Parrish & Quataert 2010) and so on, where weakly collisional plasma consists mainly of electrons and ions (protons) and high energy cosmic rays are present. Therefore, our model and results are applicable to these astrophysical objects. The main point of this investigation is finding that the back-reaction of magnetized cosmic rays can give rise to instabilities, of which the growth rate is much larger than that obtained earlier (Bell 2004). Although, the kinetic derivation of the dispersion relation in the last paper and by Zweibel (2003) contains the dynamics of cosmic rays, the contribution of the latter to the dispersion relation is absent in the hot regime. We obtain the same result for unmagnetized cosmic rays in the fluid approximation.

7. CONCLUSION

Using the multi-fluid approach, we have investigated streaming instabilities of the electron-ion plasma with relativistic and ultra-relativistic cosmic rays in the background magnetic field. Cosmic rays have been assumed to drift along the latter. The return current of the background plasma in equilibrium has been taken into account. One-dimensional perturbations parallel to the magnetic field have been considered. We have derived dispersion relations for the transverse and longitudinal perturbations, whose electric field is polarized across and along the magnetic field, respectively. We have shown that the back-reaction of magnetized cosmic rays in transverse perturbations can result in instability with the growth rate much larger than that for the Bell instability. For unmagnetized cosmic rays, we have obtained the growth rate, which is the same as the Bell's, but for the cold cosmic ray regime along the magnetic field when perturbation wavelengths are larger than the Larmor radius defined via the longitudinal momentum. For longitudinal perturbations, we have found an instability in the case of ultra-relativistic cosmic rays. The corresponding growth rate is less than that for transverse perturbations.

The results obtained can be applied to investigation of astrophysical objects such as supernova remnant shocks, galaxy clusters, intracluster medium, and so on.

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A. APPENDIX

A.1. Perturbed Velocities of Ions and Electrons

Let us put in Equation (1) $\mathbf{v}_j = \mathbf{v}_{j0} + \mathbf{v}_{j1}$, $p_j = p_{j0} + p_{j1}$, $\mathbf{E} = \mathbf{E}_0 + \mathbf{E}_1$, $\mathbf{B} = \mathbf{B}_0 + \mathbf{B}_1$. We assume that the medium and background velocities of species are uniform. Then for perturbations depending only on the z -coordinate and $\mathbf{v}_{j0} \parallel \mathbf{z}$, where \mathbf{z} is the unit vector

along the z -axis, the linearized Equation (1) takes the form

$$D_{tj}\mathbf{v}_{j1} = -\frac{\nabla T_{j1}}{m_j} - \frac{T_{j0}\nabla n_{j1}}{m_j n_{j0}} + \mathbf{F}_{j1} + \frac{q_j}{m_j c}\mathbf{v}_{j1} \times \mathbf{B}_0, \quad (\text{A1})$$

where we have used that $p_{j1} = n_{j0}T_{j1} + n_{j1}T_{j0}$ ($n_j = n_{j0} + n_{j1}$, $T_j = T_{j0} + T_{j1}$) and introduced the notations $D_{tj} = \partial/\partial t + v_{j0}\partial/\partial z$ and

$$\mathbf{F}_{j1} = \frac{q_j}{m_j}\mathbf{E}_1 + \frac{q_j}{m_j c}\mathbf{v}_{j0} \times \mathbf{B}_1. \quad (\text{A2})$$

From Equation (A1), we find equations for the perturbed transverse velocities $v_{j1x,y}$

$$\begin{aligned} (D_{tj}^2 + \omega_{cj}^2) v_{j1x} &= \omega_{cj} F_{j1y} + D_{tj} F_{j1x}, \\ (D_{tj}^2 + \omega_{cj}^2) v_{j1y} &= -\omega_{cj} F_{j1x} + D_{tj} F_{j1y}, \end{aligned} \quad (\text{A3})$$

where $\omega_{cj} = q_j B_0 / m_j c$ is the cyclotron frequency. The equation for the perturbed longitudinal velocity v_{j1z} is given by

$$\left(D_{tj}^2 - \frac{T_{j0}}{m_j} \frac{\partial^2}{\partial z^2} \right) v_{j1z} = -\frac{1}{m_j} D_{tj} \frac{\partial T_{j1}}{\partial z} + D_{tj} F_{j1z}, \quad (\text{A4})$$

where we have used the linearized continuity equation (2).

A.2. Perturbed Temperatures of Ions and Electrons

From the linearized equations (3) and (4), we obtain equations for the perturbed temperatures of ions and electrons, $T_{i,e1}$. We will assume that the background ion and electron temperatures are equal to each other, $T_{i0} = T_{e0} = T_0$. In this case, the terms connected with the perturbation of thermal energy exchange frequency in Equations (3) and (4) will be absent. However for convenience of calculations to follow the symmetric contribution of ions and electrons, we formally retain different notations for the ion and

electron temperatures. Then, we will have

$$\begin{aligned} D_i T_{i1} - \Omega_{ie} T_{e1} &= -(\gamma - 1) T_{i0} \frac{\partial v_{i1z}}{\partial z}, \\ D_e T_{e1} - \Omega_{ei} T_{i1} &= -(\gamma - 1) T_{e0} \frac{\partial v_{e1z}}{\partial z}. \end{aligned} \quad (\text{A5})$$

Here, the following notations are introduced:

$$\begin{aligned} D_i &= D_{ti} + \Omega_{ie}, D_e = D_{te} + \Omega_\chi + \Omega_{ei}, \\ \Omega_{ie} &= \nu_{ie}^\varepsilon(n_{e0}, T_{e0}), \Omega_{ei} = \nu_{ei}^\varepsilon(n_{i0}, T_{e0}), \\ \Omega_\chi &= -(\gamma - 1) \frac{\chi_{e0}}{n_{e0}} \frac{\partial^2}{\partial z^2}, \end{aligned} \quad (\text{A6})$$

where we have used Equation (5) for obtaining Ω_χ . Solutions of Equation (A5) for $T_{i,e1}$ are given by

$$\begin{aligned} DT_{i1} &= -D_e (\gamma - 1) T_{i0} \frac{\partial v_{i1z}}{\partial z} - \Omega_{ie} (\gamma - 1) T_{e0} \frac{\partial v_{e1z}}{\partial z}, \\ DT_{e1} &= -D_i (\gamma - 1) T_{e0} \frac{\partial v_{e1z}}{\partial z} - \Omega_{ei} (\gamma - 1) T_{i0} \frac{\partial v_{i1z}}{\partial z}, \end{aligned} \quad (\text{A7})$$

where

$$D = D_i D_e - \Omega_{ie} \Omega_{ei}. \quad (\text{A8})$$

A.3. Equations for Longitudinal Velocities v_{i1z} and v_{e1z}

Let us substitute Equation (A7) into Equation (A4) written for the ions and electrons. Then, we obtain

$$\begin{aligned} L_{1i} v_{i1z} + L_{2i} v_{e1z} &= DD_{ti} F_{i1z}, \\ L_{1e} v_{e1z} + L_{2e} v_{i1z} &= DD_{te} F_{e1z}. \end{aligned} \quad (\text{A9})$$

Here, we have introduced notations

$$\begin{aligned}
L_{1i} &= DD_{ti}^2 - \frac{T_{i0}}{m_i} D \frac{\partial^2}{\partial z^2} - (\gamma - 1) \frac{T_{i0}}{m_i} D_{ti} D_e \frac{\partial^2}{\partial z^2}, \\
L_{1e} &= DD_{te}^2 - \frac{T_{e0}}{m_e} D \frac{\partial^2}{\partial z^2} - (\gamma - 1) \frac{T_{e0}}{m_e} D_{te} D_i \frac{\partial^2}{\partial z^2}, \\
L_{2i} &= -(\gamma - 1) \frac{T_{e0}}{m_i} D_{ti} \Omega_{ie} \frac{\partial^2}{\partial z^2}, \\
L_{2e} &= -(\gamma - 1) \frac{T_{i0}}{m_e} D_{te} \Omega_{ei} \frac{\partial^2}{\partial z^2}.
\end{aligned} \tag{A10}$$

Solutions of Equation (A9) are the following:

$$\begin{aligned}
v_{i1z} &= \frac{D}{L} (L_{1e} D_{ti} F_{i1z} - L_{2i} D_{te} F_{e1z}), \\
v_{e1z} &= \frac{D}{L} (L_{1i} D_{te} F_{e1z} - L_{2e} D_{ti} F_{i1z}),
\end{aligned} \tag{A11}$$

where

$$L = L_{1i} L_{1e} - L_{2i} L_{2e}. \tag{A12}$$

A.4. Expressions for Perturbed Velocities via \mathbf{E}_1

Using Equation (9), we can find the components of \mathbf{F}_{j1} given by Equation (A2). In the case under consideration, we obtain

$$\begin{aligned}
F_{j1x,y} &= \frac{q_j}{m_j} D_{tj} \left(\frac{\partial}{\partial t} \right)^{-1} E_{1x,y}, \\
F_{j1z} &= \frac{q_j}{m_j} E_{1z}.
\end{aligned} \tag{A13}$$

Substitution of Equation (A13) into Equations (A3) and (A11) gives

$$\begin{aligned}
(D_{tj}^2 + \omega_{cj}^2) v_{j1x} &= \frac{q_j}{m_j} \omega_{cj} D_{tj} \left(\frac{\partial}{\partial t} \right)^{-1} E_{1y} + \frac{q_j}{m_j} D_{tj}^2 \left(\frac{\partial}{\partial t} \right)^{-1} E_{1x}, \\
(D_{tj}^2 + \omega_{cj}^2) v_{j1y} &= -\frac{q_j}{m_j} \omega_{cj} D_{tj} \left(\frac{\partial}{\partial t} \right)^{-1} E_{1x} + \frac{q_j}{m_j} D_{tj}^2 \left(\frac{\partial}{\partial t} \right)^{-1} E_{1y}
\end{aligned} \tag{A14}$$

and

$$\begin{aligned} v_{i1z} &= \frac{D}{L} \left(L_{1e} D_{ti} \frac{q_i}{m_i} - L_{2i} D_{te} \frac{q_e}{m_e} \right) E_{1z}, \\ v_{e1z} &= \frac{D}{L} \left(L_{1i} D_{te} \frac{q_e}{m_e} - L_{2e} D_{ti} \frac{q_i}{m_i} \right) E_{1z}. \end{aligned} \quad (\text{A15})$$

A.5. Perturbed Plasma Current

The components of the transverse perturbed plasma current $j_{pl1x,y} = \sum_j q_j n_{j0} v_{j1x,y}$ are found by using Equation (A14). The expressions for $4\pi (\partial/\partial t)^{-1} j_{pl1x,y}$ can be given in the form

$$\begin{aligned} 4\pi \left(\frac{\partial}{\partial t} \right)^{-1} j_{pl1x} &= \varepsilon_{plx} E_{1x} + \varepsilon_{plx} E_{1y}, \\ 4\pi \left(\frac{\partial}{\partial t} \right)^{-1} j_{pl1y} &= -\varepsilon_{plx} E_{1x} + \varepsilon_{plx} E_{1y}, \end{aligned} \quad (\text{A16})$$

where $\omega_{pj} = (4\pi n_{j0} q_j^2 / m_j)^{1/2}$ is the plasma frequency. The following notations are introduced in Equation (A16):

$$\begin{aligned} \varepsilon_{plx} &= \sum_j \frac{\omega_{pj}^2 D_{tj}^2}{(D_{tj}^2 + \omega_{cj}^2)} \left(\frac{\partial}{\partial t} \right)^{-2}, \\ \varepsilon_{plx} &= \sum_j \frac{\omega_{pj}^2 \omega_{cj} D_{tj}}{(D_{tj}^2 + \omega_{cj}^2)} \left(\frac{\partial}{\partial t} \right)^{-2}. \end{aligned} \quad (\text{A17})$$

The longitudinal perturbed plasma current $j_{pl1z} = \sum_j q_j n_{j0} v_{j1} + \sum_j q_j n_{j1} v_{j0}$ is found by using Equations (2) and (A15)

$$\begin{aligned} 4\pi \left(\frac{\partial}{\partial t} \right)^{-1} j_{pl1z} &= \frac{\omega_{pi}^2}{D_{ti}} \frac{D}{L} \left(L_{1e} D_{ti} - L_{2i} D_{te} \frac{q_e m_i}{q_i m_e} \right) E_{1z} \\ &+ \frac{\omega_{pe}^2}{D_{te}} \frac{D}{L} \left(L_{1i} D_{te} - L_{2e} D_{ti} \frac{q_i m_e}{q_e m_i} \right) E_{1z}. \end{aligned} \quad (\text{A18})$$

B. APPENDIX

B.1. Perturbed Velocity of Cosmic Rays

The linearized version of Equation (6) for $\mathbf{p}_{cr1} = \mathbf{p}_{cr} - \mathbf{p}_{cr0}$ and $\mathbf{v}_{cr0} \parallel \mathbf{z}$ has the form

$$R_{cr0}D_{tcr}\mathbf{p}_{cr1} + \mathbf{p}_{cr0}D_{tcr}R_{cr1} = -\frac{\nabla p_{cr1}}{n_{cr0}} + m_{cr}\mathbf{F}_{cr1} + \frac{q_{cr}}{c}\mathbf{v}_{cr1} \times \mathbf{B}_0, \quad (\text{B1})$$

where $D_{tcr} = \partial/\partial t + v_{cr0}\partial/\partial z$ and

$$m_{cr}\mathbf{F}_{cr1} = q_{cr}\mathbf{E}_1 + \frac{q_{cr}}{c}\mathbf{v}_{cr0} \times \mathbf{B}_1. \quad (\text{B2})$$

For the perturbed transverse velocities of cosmic rays $v_{cr1x,y}$, we find from Equation (B1) the following solutions:

$$\begin{aligned} (D_{cr}^2 + \omega_{ccr}^2) v_{cr1x} &= \frac{q_{cr}}{m_{cr}}\omega_{ccr}D_{tcr} \left(\frac{\partial}{\partial t}\right)^{-1} E_{1y} + \frac{q_{cr}}{m_{cr}}D_{cr}D_{tcr} \left(\frac{\partial}{\partial t}\right)^{-1} E_{1x}, \\ (D_{cr}^2 + \omega_{ccr}^2) v_{cr1y} &= -\frac{q_{cr}}{m_{cr}}\omega_{ccr}D_{tcr} \left(\frac{\partial}{\partial t}\right)^{-1} E_{1x} + \frac{q_{cr}}{m_{cr}}D_{cr}D_{tcr} \left(\frac{\partial}{\partial t}\right)^{-1} E_{1y}, \end{aligned} \quad (\text{B3})$$

where $D_{cr} = \gamma_{cr0}R_{cr0}D_{tcr}$. When obtaining Equation (B3), we have expressed $F_{cr1x,y}$ through $E_{1x,y}$ by using Equation (A13) for $j = cr$ (see Equation (B2)).

The z -component of Equation (B1) is given by

$$D_{cr}v_{cr1z} + v_{cr0}R_{cr0}D_{tcr}\gamma_{cr1} + \gamma_{cr0}v_{cr0}D_{tcr}R_{cr1} = -\frac{1}{m_{cr}n_{cr0}}\frac{\partial p_{cr1}}{\partial z} + F_{cr1z}, \quad (\text{B4})$$

where $\gamma_{cr1} = \gamma_{cr0}^3 v_{cr0} v_{cr1z} / c^2$.

B.2. Perturbed Temperature and Pressure of Cosmic Rays

We now find R_{cr1} and p_{cr1} . From Equation (8), we see that

$$R_{cr1} = \frac{\Gamma_{cr}}{\Gamma_{cr} - 1} \frac{T_{cr1}}{m_{cr}c^2}. \quad (\text{B5})$$

The perturbation of the temperature T_{cr1} found from the equation $T_{cr} = \gamma_{cr} p_{cr} / n_{cr}$ is equal to

$$T_{cr1} = T_{cr0} \left(\frac{p_{cr1}}{p_{cr0}} - \frac{n_{cr1}}{n_{cr0}} + \frac{\gamma_{cr1}}{\gamma_{cr0}} \right). \quad (\text{B6})$$

From Equation (7), we can find the pressure perturbations p_{cr1}

$$p_{cr1} = p_{cr0} \Gamma_{cr} \left(\frac{n_{cr1}}{n_{cr0}} - \frac{\gamma_{cr1}}{\gamma_{cr0}} \right), \quad (\text{B7})$$

where

$$n_{cr1} = -n_{cr0} \frac{\partial v_{cr1z}}{D_{tcr} \partial z}. \quad (\text{B8})$$

B.3. Equation for v_{cr1z}

Substituting Equations (B5)-(B8) into Equation (B4) and using Equation (A13) for cosmic rays, we obtain

$$\gamma_{cr0}^3 \left(R_{cr0} - \frac{\Gamma_{cr} T_{cr0}}{m_{cr} c^2} \frac{v_{cr0}^2}{c^2} \right) D_{tcr}^2 v_{cr1z} - 2\gamma_{cr0} v_{cr0} \frac{\Gamma_{cr} T_{cr0}}{m_{cr} c^2} D_{tcr} \frac{\partial v_{cr1z}}{\partial z} - \frac{\Gamma_{cr} T_{cr0}}{\gamma_{cr0} m_{cr}} \frac{\partial^2 v_{cr1z}}{\partial z^2} = D_{tcr} \frac{q_{cr}}{m_{cr}} E_{1z}. \quad (\text{B9})$$

B.4. Perturbed Cosmic Ray Current

The perturbed transverse cosmic ray currents $j_{cr1x,y} = q_{cr} n_{cr0} v_{cr1x,y}$ are found by using Equation (B3). For the values $4\pi (\partial/\partial t)^{-1} j_{cr1x,y}$, we will have

$$\begin{aligned} 4\pi \left(\frac{\partial}{\partial t} \right)^{-1} j_{cr1x} &= \varepsilon_{crxx} E_{1x} + \varepsilon_{crxy} E_{1y}, \\ 4\pi \left(\frac{\partial}{\partial t} \right)^{-1} j_{cr1y} &= -\varepsilon_{crxy} E_{1x} + \varepsilon_{crxx} E_{1y}. \end{aligned} \quad (\text{B10})$$

Here

$$\begin{aligned}\varepsilon_{crxx} &= \frac{\omega_{pcr}^2 D_{cr} D_{tcr}}{(D_{cr}^2 + \omega_{ccr}^2)} \left(\frac{\partial}{\partial t} \right)^{-2}, \\ \varepsilon_{crxy} &= \frac{\omega_{pcr}^2 \omega_{ccr} D_{tcr}}{(D_{cr}^2 + \omega_{ccr}^2)} \left(\frac{\partial}{\partial t} \right)^{-2},\end{aligned}\tag{B11}$$

where $\omega_{pcr} = (4\pi n_{cr0} q_{cr}^2 / m_{cr})^{1/2}$ is the cosmic ray plasma frequency.

The longitudinal perturbed cosmic ray current is equal to $j_{cr1z} = q_{cr} n_{cr0} v_{cr1z} + q_{cr} n_{cr1} v_{cr0}$. Making use of the linearized continuity equation for cosmic rays and Equation (B9), we find

$$4\pi \left(\frac{\partial}{\partial t} \right)^{-1} j_{cr1z} = \frac{\omega_{pcr}^2}{L_{cr}} E_{1z},\tag{B12}$$

where

$$L_{cr} = \gamma_{cr0}^3 \left(R_{cr0} - \frac{\Gamma_{cr} T_{cr0}}{m_{cr} c^2} \frac{v_{cr0}^2}{c^2} \right) D_{tcr}^2 - 2\gamma_{cr0} v_{cr0} \frac{\Gamma_{cr} T_{cr0}}{m_{cr} c^2} D_{tcr} \frac{\partial}{\partial z} - \frac{\Gamma_{cr} T_{cr0}}{\gamma_{cr0} m_{cr}} \frac{\partial^2}{\partial z^2}.\tag{B13}$$